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# SCEPTICISM—CREDULITY=0

# THE RELIGION OF ALGEBRAIC CURVES

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MDCCCLXXXII

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# SCEPTICISM − CREDULITY = 0.

## THE RELIGION OF ALGEBRAIC CURVES.

MODERN scientific sceptics urge fairly enough from their special point of view, that religion should be examined by the cold light of science, and that all that cannot stand the hard test of reasoning and of evidence, should be consigned to the limbo of superstition. They rightly deny any scientific weight to mere sensational emotions, which unfortunately constitute so large a share of modern religion, and which are often as dependent on the state of a man's health, as on the state of his soul. The champions of orthodoxy, on the other hand, frequently fall into the error of defending their opinions, on the very ground most unfavourable to themselves. They often appeal by preference to personal experiences and feelings, and sternly repudiate any mathematical reasoning, from not appreciating the fact, that it is precisely the most unfavourable battle-ground for scepticism. Viewed mathematically, scepticism appears in its true light as the very highest

development of credulity. Sceptics consequently encourage the idea, that religion is *ipso facto* beyond the sphere of mathematics; whilst marvelling at the blindness which, in military parlance, induces the troops of orthodoxy to stand on the defensive, in comparatively indefensible positions, instead of assuming the offensive against that weak flank of scepticism — the purely mathematical view of religion, summed up in the equation

$$\text{Scepticism} - \text{Credulity} = 0.$$

To define scepticism as the very highest development of credulity sounds paradoxical, but it is not more so than the proper definition of a scientific expert—he who best knows his scientific ignorance. Take a hoop, and saw it across at one point. Call the narrow space between the saw-cut, truth; its edges, knowledge and ignorance.

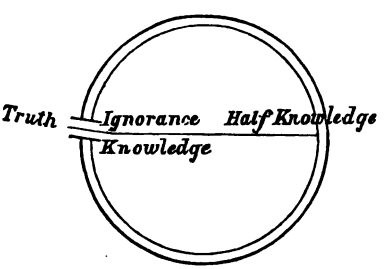



Fig. 1.

It is a long continuous path along the hoop, from edge to edge of the cut; and the point in the path farthest from

the truth, is also that which is the farthest from ignorance as from knowledge, that half-knowledge which the intellectual progress of the nineteenth century may perhaps be approaching, that scepticism which, being the farthest from the truth, is therefore the highest development of credulity.

Profiting by any admission made by believers, that a personal God, being the originator of all law, must be above the law, and that in the relations between the moral and physical world, mathematical reasoning is inadmissible, as necessarily limiting the power of the Deity, sceptics push their advantage by asking, "All the physical universe is governed by scientific, *i.e.* mathematical, laws. How, then, can these laws be altered for your special benefit?" They appeal to Cæsar's laws against Cæsar, to the universal lawgiver against the cry of the mother over her dying babe; and as nature's laws follow their relentless course, they conclude that the mathematically-governed physical universe, cannot, in the remotest degree be altered or affected, by prayer to a personal God. Which of us has not felt the force of this argument? Which of us, as generation after generation is crushed out, has not asked with the Persian poet, "Who can resist the grinding of the two millstones—the sky above, and the earth beneath?"

But strong as this argument may seem, it is, to the mathematician, a hopelessly weak one; for it is founded on an entire misconception of the nature of a mathematical law. In popular belief, a mathematical law must

be indicated by a regular, inelastic, unaltering, continuous series of effects due to certain causes. In real fact, a mathematical law may, and often does, represent in a certain sense the most irregular, elastic, variable, and discontinuous series of effects. The very fact, indeed, of the universe being directed by mathematical laws, is the only conceivable method, whereby even an infinite intelligence could have left itself an infinite latitude for deducing the most absolutely varying, contradictory, and incongruous effects, from invariable and constant causes, such as seem to be the moving agents in the system of the universe. In view of the objections to a personal God carrying out constant changes and contradictions in the mathematical laws of nature, in compliance with the prayers of individuals or nations, the mathematical line of argument would briefly be as follows: "Admitting the government of the universe by mathematical laws, and knowing that such laws may represent the most contradictory and incongruous series of effects, the *onus probandi* lies with the sceptic to show, that apparent changes and contradictions in answer to prayer would be mere acts of omnipotent will, in other words, violations of law, and would not be due to mathematical laws, involving such irregularities in obedience to law." As a matter of feeling, the believer in God's all-pervading Spirit may shrink from the thought of such a Spirit being bound by mathematical laws; but he may in argument fairly say to the sceptic, "Thou hast

appealed unto science, and unto science shalt thou go;" and find, that even when tried by such a cold, unimpassioned abstraction of science as an algebraical equation, the theory of a personal, or, to speak in all reverence, an interfering God, is not only possible, but in the highest degree probable and scientific.

But before going further, it may be well to define a mathematical law as being merely a statement of equality —an equation. Every equation can be graphically represented, and brought before the eye in a diagram. To explain the relation between an algebraic equation, and its graphic representation, an algebraic curve, we must go back to very simple considerations. A detective searching a room for a stolen paper, finds a remark in the thief's notebook, "Four from corner in length, two across;" and counting from the corner of the room four squares in the carpet one way, and two in the other, there finds the paper under the carpet. The point had been defined, with reference to the corner and to the two side walls, in terms of the carpet squares. Similarly, if we draw two indefinite straight lines or axes intersecting at right angles on a sheet of paper, we may define the exact position of any point by such a statement as "four inches in length, two inches across," with reference to the intersection of the axes, which is called the origin, by measuring four inches from it one way and two inches the other. Calling all measurements along one axis $x$,

and all measurements along the other $y$, the point is defined by the two equations

$$x=4,$$

$$y=2,$$

$x$ and $y$ being called the co-ordinates of the point as measured from the origin.

Going a step further, if from the origin we draw a

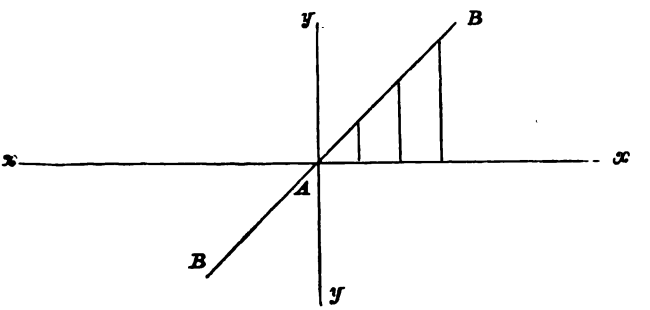



Fig. 2.

line $AB$ bisecting the right angle between the axes, it is self-evident that for every possible point in that line, the co-ordinates are always equal, and fulfil the condition

$$x=y,$$

which therefore is the equation to a straight line, bisecting the angle between the rectangular axes, and passing through the origin, the said straight line being the graphic representation of the equation

$$x=y,$$

which embodies the mathematical law binding upon

every single separate point of the line so drawn. Similarly the equation

$$x^2+y^2=r^2$$

embodies the characteristic mathematical law of, and is represented by, a circle whose radius is $r$. The equation

$$a^2y^2+b^2x^2=a^2b^2$$

is represented by an ellipse with $2a$ and $2b$ as its major and minor axes, these curves being the graphic embodiment of the mathematical laws expressed by these equations. To avoid being tedious, it may briefly be said, that every possible mathematical law, in every conceivable variety, may thus be represented algebraically by an equation, and graphically by a curve, the outline of which is made up of an infinite succession of points, each completely satisfying a special law.

The most subtle laws, without exception, of the physical universe are undoubtedly mathematically governed, and may therefore be expressed by equations and curves. But mathematicians must, in the first place, detect the laws; and, secondly, possess themselves of a calculus sufficiently powerful to crack the terribly hard nut of mere technical mathematical difficulty, in which the simplest physical problems are embedded. Such a simple problem, for example, as the continuous flow of water, has, for want of a sufficiently powerful calculus, baffled the highest mathematical skill. Modern mathematics, indeed, are like an infant king, who, being a lawful

monarch, rightly claims theoretical supremacy over, but practically neither controls nor understands, his subjects. The mathematician knows that all physical problems are, theoretically, given into his hand, but that the most powerful calculus hitherto devised by man, is unable to explain and disentangle the practical working of the overwhelming majority of such problems, even if general laws regarding them had been ascertained. What progress may be made when, in the future, the Calculus of Quaternions is looked upon much as we now regard common arithmetic, none may predict; but mathematical science is as yet but an infant, although in it is vested the hope and the right of explaining, and possibly controlling, physical phenomena.

But if mathematicians have hitherto had but moderate success in practically solving natural problems, they have learned, that just as no apparent *bizarrerie* in the graphic representation of a law or equation can be so startling, as necessarily to be incompatible with the strictest adherence to law, so no apparent change in a mathematically-governed phenomenon can be scientifically pronounced, from *a priori* observation, to be an arbitrary act of will, or violation of law, unless its complete law is known from an algebraic equation; and unless its curve, in mathematical language, is completely traceable from positive to negative infinity, from the infinity of the future to the infinity of the past. They see, for example, that a practised algebraist can imagine a mathematical law, can clothe it in an

equation, and reproduce it graphically in such a curve as shown below, which may further be but one branch out of as many equally fantastic branches as the degree of the equation. In other words, if the equation were

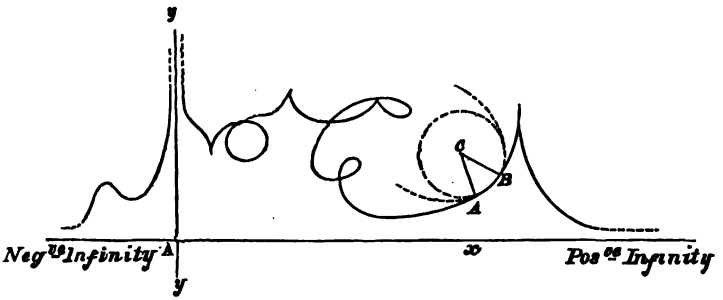



Fig. 3.

of the fifth or fiftieth degree, there would be five or fifty, distinct, similar, or dissimilar, branches all satisfying one and the same law.

Mathematically speaking, all mere *a priori* reasoning, all mere appeal to past experience of natural phenomena, is as unscientific as the assumption, that, because a portion of the above curve from *A* to *B* coincides as closely as human observation can appreciate, with a portion of a finite circle, that therefore a body, really moving along the erratic infinite curve, is moving along a finite circle, or along an infinite parabola, where practically coinciding with thirty degrees of the circular arc.

Now the overwhelming majority of physical problems, are not yet defined by any certain mathematical laws. Even

with respect to such a well-known law as that of gravitation, let us assume it, by way of illustration, to be represented by the circle *ABC* described in the last figure. The nature of an algebraic curve of which a certain number of points are accurately determined can always be found; but a portion *AB* of an irregular curve may be so nearly circular, that the difference cannot, by any possibility be detected by mere observation. Further, our actual observations on gravitation may perhaps have extended, as to time and space, over what may only be an infinitesimal portion of the arc *AB*, which itself again, is but an infinitesimal portion of the whole curve, extending from the infinite past to the infinite future. To argue that a violation of law is necessarily involved by any irregularity in the movements of bodies governed by gravitation may therefore, mathematically, be as incorrect as to conclude, that the erratic curve shown above is a circle, because an infinitesimal part of an infinitesimal portion, such as *AB*, of its whole length is found, by confessedly imperfect observations and instruments, to coincide very nearly with a circle. All astronomical observations, for example, based on angular measurement, are evidently at best but fair approximations. What enormous differences between the real and apparent positions of heavenly bodies, would arise from the most infinitesimal deviation in the direction of the plumb-line, whether due to undetected local attraction, or to those

earth tremors which, like daily tides, seem to sweep over the earth's surface!

To mark even more strongly the weakness, mathematically speaking, of the otherwise powerful argument, against miracles or special interventions, founded on *a priori* reasoning and past experience, it should further be remembered that, even when the equation to a curve is given, two distinct processes are required before it can be completely traced; and before any abnormal appearance in its graphic representation, can be declared to be a departure from law. Confining ourselves to geometry of two dimensions, a curve can be traced by points, by deducing one variable from assumed successive values of the other. This process, with an equation above the second order, is generally a process of approximation which, when the equation is a complicated or transcendental one of a high order, is so laborious, as to be practically useless unless to determine a very few points in a possibly infinite curve. This purely algebraic method is however quite powerless to detect, except by the merest chance, the points at which, by law, there are what may be called miracle points in the graphic delineation of the curve; the points where it goes back on itself, or becomes discontinuous, or otherwise presents an abnormal character. In the curves below, forming the positive and negative branches of one equation, we might thus trace the branches by points, say, from $A$ to $B$, or from $C$ to $D$; but we could

not find the exact so-called miracle points *H*, *E*, *F*, *G*, where the positive branch passes from concave to convex,

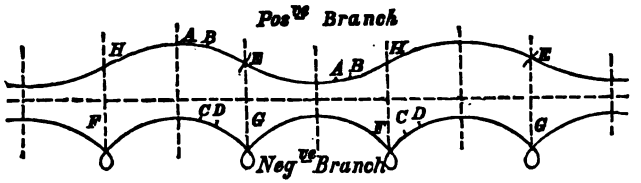



Fig. 4.

and where the negative branch returns in loops on itself. To do this, the differential calculus must be employed, which, although using algebra as its tool, is distinctly not an algebraic process.

Hence, before it can be asserted with any show of mathematical truth that a so-called miracle is impossible, as contrary to law, we must be possessed, with respect to the moral or physical phenomena bearing on the miracle, (1) of the whole and exact, and not merely approximate, law; (2) of the power, corresponding to the algebraic process of tracing by points, of fixing the normal course of that law at known points, or in other words, under known circumstances; (3) of the power, corresponding to the processes of differentiation, of detecting the unknown points, or unknown circumstances, at which, or under which, the absence of the miracle would actually be a violation of the law. Can any moral or natural philosopher for an instant claim to be possessed of all, or even of one, of these

laws and powers, without making himself supremely ridiculous as the embodiment of the very highest development of credulity in his own intellectual powers?

Supposing that our philosophers are acquainted with even a very small proportion of the physical forces at work in the universe (which is very doubtful), and admitting that such forces may very possibly, sooner or later, be traced up to one centre of force, the fact still remains, that we at present possess no law, regarding their correlation, which we can practically apply to some of the simplest physical problems. The practical engineer asks, why the chemical action of an infinitesimal portion of gold in steel manufacture, increases the tensile strength in much the same way as the gradual abstraction of caloric by tempering in hot oil. The practical electrician asks some such other question; and they are both very willing indeed to avail themselves of what the *savant* may have to say about it. But their practical common-sense revolts, when he poses as the dogmatic arbiter of the possible, and the impossible, in physical problems. When we come to phenomena where moral and physical causes act simultaneously, as in the case of the epileptic girl who turned somersaults into the air for hours together, thus developing enormous mechanical power drawn from no appreciable source, or as in the case, witnessed by the writer, of a frail Hindoo priest of seventy years of age, dancing and springing about under a kind of demoniac possession for more than an hour, whilst

swinging an iron trident which a strong man could barely lift, all the data for scientific dogmatism are absolutely deficient. Indulging again in a mathematical analogy, and referring back to fig. 4, the physical aspects of such phenomena may, for want of better data, be fairly and profitably discussed by the light of past experience, much as the shape of the positive or physical branch of the curve, from *B* to *H*, may be roughly guessed at, if its shape from *A* to *B* be previously known. But we have no right whatever, from anything we may observe of the shape of the positive or physical branch, to draw any conclusions as to the shape of the corresponding portion of the negative or moral branch. Much less have we any right to deny the existence of such a branch; or to doubt that, though dissimilar and separate, the two branches, as represented by one equation, embody one and the same harmonious physico-moral law.

But the moment we speak of physico-moral laws, and hint at the possibility of the moral world being quasi-mathematically governed, an objection suggests itself to the disbeliever in a personal, prayer-hearing, God. He at once says, "The mathematical view may, by analogy, tell against objections made to physical miracles as necessarily departures from law; but if the moral world be also mathematically governed, that view merely leads from the Scylla of scepticism, into the Charybdis of fatalism. What becomes of the freewill both of God and of man?" The

answer to this objection is perfectly simple. A reasoning man may be compelled, by law, to pass from Liverpool to New York, but with the complete option of selecting his own route; an unreasoning electrical current may also, by law, be compelled to do the same thing, but only by forcing it to keep to a defined, insulated cable. In one case freewill is exerted, in the other it is not; but in both cases, obedience to law may be unavoidable. In mathematical language, reasoning man is given moral laws, expressed by equations, with full liberty to select their arbitrary constants; unreasoning matter is given physical laws, expressed by equations, in which the arbitrary constants have been fixed beforehand. Analogies may be pushed too far, but this mathematical view bears so strongly upon this question of freewill, that a numerical illustration of arbitrary constants may not be out of place.

We know, for example, that all algebraic equations of the second order represent some sort of conic section, a circle or ellipse, a parabola or a hyperbola. A master may give his pupil a complete numerical equation of the second order, such as

$$3x^2 - 4xy + 3y^2 + y - x - 9 = 0,$$

and tell him to trace its graphic delineation. The pupil will determine (1) that it is a conic section, (2) that it is an ellipse, (3) that it is an ellipse, with 4 as its major and $\frac{4}{\sqrt{5}}$ as its minor axis. No freewill whatever has been left to the pupil. In order to satisfy the master's

law, he must trace that particular ellipse and no other. Again, the master gives him the general equation of the second order, necessarily also a conic section,

$$ax^2+bxy+cy^2+dx+ey+f=0,$$

and tells him he may himself select numerical values for the arbitrary constants $a$, $b$, $c$, $d$, $e$, $f$, and trace the resulting curve. He finds that, (1) as before, his freewill is limited to the extent of tracing a conic section of some sort; (2) exercising his freewill in assigning values to the arbitrary constants, he may trace any conic section of any possible shape or size, and still completely satisfy the master's law. As in these two cases, the physical forces of the universe may depend on numerical equations expressing general fixed laws, the moral forces may depend upon general equations also involving general laws, but modified in endless variety by the freewill of reasoning man.

To assert, that the moral and physical world may be linked together by some such laws as connect the positive and negative branches of the same algebraic curve, would not be such a wild flight of imagination as is indulged in by modern philosophers who trace back all life to a protoplasm, or, better still, to meteoric dust or to a problematical moss-grown stone from another planet. If a small scientific minority can so far indulge their fancies, why tax with credulity those who, guided by mathematical analogy, believe that as every point in the negative branch of the curve in fig. 4 has its corresponding

point in the apparently disconnected positive branch, so every prayer has, by law, its echo in the physical world? Just as cusps in the positive, so often correspond to multiple points in the negative branch, so special physical miracles may, by law, answer to special conditions of prayer. But however rational such a belief may be, it is not necessary to be held, in order to understand how prayer may fit into, and correspond with, physical phenomena. The flexibility introduced into moral laws by freewill—*i.e.* by the free selection of arbitrary constants—as contrasted with the rigidity of physical laws—*i.e.* the unalterably fixed constants—is in itself sufficient to account for their harmonious working, like plastic clay fitting into an iron mould, without necessarily being linked together by one common law.

But prayer may be considered from another point of view, which, although it may not commend itself to disbelievers in the Spirit of God indwelling in man, will appeal with great force to the vast numbers with whom prayer on every conceivable subject, and in every conceivable circumstance, is as natural an impulse, as that which draws the little child to its mother in its everyday life and difficulties. To apply a mathematical analogy to such a feeling, we may suppose that, in the last-mentioned case, the master tells the pupil that, whilst leaving him perfectly free to select for himself the constants $a, b, c, d, e, f$, he, the master, will, if asked, explain the conditions which

must exist between them in order to represent an ellipse, parabola, or hyberbola, under the general condition of the curve being a conic section; and will further suggest, if asked, how they should be selected, so as to produce the special curve most suitable to any given case. If the pupil either neglects to ask to be informed of these conditions, or ignores them when pointed out to him, he may run the risk of selecting the wrong curve, and of coming into collision with some unyielding physical law. In Christ's words, "Ask, and ye shall receive," the Christian hears the Master's offer of assistance; the Holy Spirit defines to his conscience the conditions of right and wrong; and in prayer he secures his Father's guidance in the smallest details of everyday life. Viewed mathematically, the Christian in so acting is more scientific than the sceptic; for he trusts less to chance, and thereby more fully acknowledges scientific laws.

But when conscience is mentioned, the sceptic will say, "No two consciences agree as to what is right and what is wrong. It is all a mere matter of habit and of training, for if inspired by one Spirit, all consciences would agree." In taking this view, the sceptic unconsciously re-echoes the narrow dogmatism, which would stamp one image upon all religious thought. The Christian, on the other hand, sees in the world fathers and mothers guiding the consciences of their children, according to each child's individual disposition. Christ has promised him guidance; and his conscience hears the Holy Ghost, the Comforter's teaching, with as little thought of imposing narrow conditions upon

it, as the little child encouraged to cross the stepping-stones of the brook, whilst the mother follows close and her voice gently warns. He knows, like the child, how easily he falls, but believes that a strong hand will somehow put him on his feet, without needing to be told how it should exactly be done. Whilst shrinking from laying down the law to other men's consciences, he tries to act up to Abraham Lincoln's noble saying, "With malice towards none, with charity towards all, let each man do the right as God has given him to see the right." This is the Christian's perhaps unscientific view of conscience; but if he be challenged to take a purely scientific and mathematical view of the matter, the analogies of envelope curves and diametral curves, will illustrate how conscientious views in the present life may vary indefinitely, by the action of freewill, without infringing the general laws laid down for man's guidance; and how such diverging views may, in the life to come, all merge into the perfect will of God.

Envelope and diametral curves present endless analogies bearing on this view; but to refer to envelope curves, would require technicalities which would here be out of place. The following example of a diametral curve, taken almost at haphazard, will illustrate why, to a mathematician, the theory is so untenable, that because consciences do not all agree, they cannot all be inspired by one Spirit, but merely depend on habit and training. The curve is represented by the equation

$$y = ax^2 + c\sqrt{x}\sin bx,$$

in which $a$, $b$, and $c$ are arbitrary constants. Let us suppose, so as to conform to the analogy previously discussed as between master and pupil, that the former, whilst laying down the general law expressed in the equation, himself selects the constant $a$, whilst the latter is at liberty to choose the constants $b$ and $c$. The curve when traced has the form shown below.

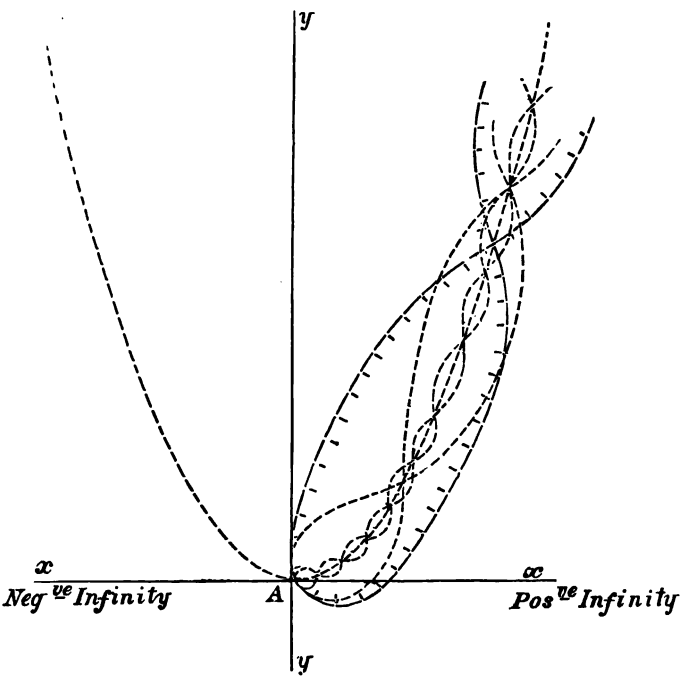



Fig. 5.

When $x$ is positive—*i.e.* when the abscissæ are measured to the right of $A$—the two branches of the curve forming loops alternately recede from, and approach, a diametral curve shown by black dots, which is the parabola whose equation is $y=ax^2$; meeting it whenever sin $bx=o$, or whenever $bx$ is a multiple of $\pi$. When $x$ is negative—*i.e.* when the abscissæ are measured to the left of $A$—$y$ is impossible, except when sin $bx=o$, in which case $y=ax^2$; so that, on the negative side, there are an infinite number of conjugate points, each one situated on the parabola, over against a double point in the positive branches, the successive abscissæ being $\frac{\pi}{b}$, $\frac{2\pi}{b}$, $\frac{3\pi}{b}$, etc. The greater the value of $b$, and the less the value of $c$, the less do the loops on the positive side diverge from the diametral curve, and the oftener do they intersect on, and coincide with, it. Further, the greater the value of $b$, the closer do the points on the negative side approach each other; and if $b$ were infinitely great, they would actually form a branch of the parabola whose equation is $y=ax^2$.

Now what analogies does the graphic representation of this curious curve suggest to the mathematician? Let the present life be the positive, and the life to come the negative, side of the curve. The Master, God, wills the general law as expressed by the complete equation; and then by fixing the constant $a$, defines His special perfect

law, from which in the life to come there may be no possible divergence, and which in the present life is the diametral or leading curve, along which, although not actually a branch of the curve, all consciences must group themselves if they conform to the general universal law; the law which St. Paul so strongly insists on in the opening chapters of the Epistle to the Romans. The pupil, man, is allowed complete freewill in the selection of the constants $b$ and $c$, which leaves him the utmost latitude of conscience, without in any way interfering with the diametral curve. According to this freewill selection, which may no doubt be influenced by time, education, locality, etc., each man's conscience will guide him in a path, approaching or receding from the diametral curve in compliance with the general law, and subject only to the condition that, in certain cases, (corresponding to the double points in the loops) the law of his conscience, if obeyed, must lead to a point of coincidence, of complete accordance with, the leading curve of God's perfect law. We see how, in the figure, the different loops intersect the parabola at totally different points, and how the selection of the constants $b$ and $c$ affects the number of intersections with, and amount of divergence from, the diametral curve. If $b$ be very large and $c$ very small, the loops will almost coincide with the parabola diverging inappreciably from it, between numberless close intersections; if $b$ be very small and $c$ very large, the loops will diverge largely from, and only meet the parabola at,

a very few distant points. Precisely in the same way, may the course of action of two men in similar circumstances be widely different, whilst they both follow God's law and obey their consciences. If, however, one man submits himself more completely than the other to the guidance of the Holy Spirit, he will, speaking in all reverence, be taught, in answer to prayer, to select proper values for the constants $b$ and $c$; and in consequence will conform more closely to, and be oftener in complete harmony with, the perfect law of God. Again, a man from want of knowledge may follow the teachings of natural religion, and unwittingly select such values for $b$ and $c$ that, though obeying the general universal law, he may diverge far from, and be seldom in complete harmony with, the perfect law of God. This would quite support St. Paul's saying of the Gentiles, "These having not the law, are a law unto themselves." As every double point in the positive branch, has a corresponding conjugate point in the negative branch of the curve, we may, by analogy, suppose that those who in this life least diverge from, and most frequently are in complete harmony with, the perfect law of God, will, in the life to come, be most like God; the reverse being the case, with those who, in this life, do not live very close to God, or deliberately ignore or violate the right, as God has given them to see the right. From this point of view conscience may be regarded, not merely as warning us in the present life, but almost as a self-acting engine, registering the influence

of every thought, word, and deed upon the life to come—knowing with God, or as well as God, as is suggested by the very derivation (*con-scio*) of the word.

The thought further arises, that as the parabola on the positive side is not a part of the curve, but is merely diametral to the actual branches, so complete accordance with God's perfect law may not be attainable, owing to our physical disqualifications, in the present life, although the more we watch and pray, the closer we will cling to it, the oftener we will entirely satisfy it. As the master might trace the parabola, on which the pupil might check his work in tracing the graphic delineation of the curve, so one of the objects of the incarnation of Christ, perfect God as well as perfect man, and so fulfilling completely the perfect law of God, would be to provide mankind, by teaching and example, with a gauge by which to test their consciences and their actions. Lastly, what an emphatic sermon of toleration does the curve preach to persecuting priests of every creed, to dogmatic philosophers of every school!

The scientific unsoundness of scepticism, may be further considered from the point of view of mathematical *loci*. Two circles, for example (see fig. 6), of which *A* is five times larger than *B*, are twenty-four miles apart. The dotted circumference of a circle, five miles in diameter, is described from *C* as a centre; and is the *locus* whence *A* and *B* everywhere subtend equal angles.

If angular measurement alone were the criterion of size (as is very possibly the case with infants before sight has been corrected by experience), the unequal circles as

Fig. 6.

seen from the *locus*, would always seem exactly equal. Imagine an observer, thus referring everything to angular measurement, to move along a guiding groove on the circumference of the *locus;* and we have an exact image of the possible part played, in any scientific research, by any one of our senses. Just as the perfection of the measuring instrument employed, and as skill in its use, would confirm such an observer in his erroneous estimate of size, so may the very perfection of skill, in researches dependent upon any one sense, only confirm some delusion due to some inexorable law of our being, along which, as along a guiding groove, that sense may be moving, whether or no, we like it, or know it. As in the case, where readily appreciating three dimensions, we utterly fail to grasp the idea of a fourth, we are too conscious of such enforced limitations to our faculties, to admit of our ignoring their

existence. And yet, science calmly requires us to accept, as from the final judges of all our beliefs, the verdict of senses and faculties, admittedly and helplessly moving along *loci*, the special properties of which, as affecting and biassing the truth and justice of such a verdict, science itself is just as ignorant of now, as it was in the days of Job.

It would therefore seem that the Christian's views regarding miracles, freewill, and conscience scarcely lay him open to the charge of credulity, which, mathematically speaking, may more fairly be brought against the sceptic, who, with the blunt compasses and incorrect scale of measurement supplied by his own intellect, would pronounce dogmatically upon the nature of an algebraic curve, from examining an infinitesimal-traced portion of its length, without either knowing its equation, the algebra required to approximately trace it, or the differential calculus needed to detect its singular points. When we come to higher branches of mathematics—geometry of three dimensions, differential equations, the calculus of variations, etc.—the difficulty is not to find, but to select from, such analogies. The calculus of probabilities may no doubt show that there are great odds in favour of this or of that view, but great odds do not constitute scientific proof. If the sceptic trust to probabilities, he is a mere gambler, led from a few runs of luck to stake his all, without knowing the odds in favour of the bank. If this be so, may not scepticism be fairly

defined as the highest development of credulity? In terse mathematical language, "it assumes the continuity of the function from the continuity of the variable, and *vice versâ*."

The views here expressed, and which, for a mathematician, may be described as all condensed into the last sentence, do not claim to touch more than a few points in the controversy, but may show that religion has no cause to avoid discussion, on grounds of pure mathematical reasoning. Two natural, but not well-founded objections may be made to such a mathematical view. The first objection is briefly, "What practical value and influence can cold abstractions, like algebraic equations and symbols, exercise on everyday work, life, and belief?" The reply is, that these abstractions to a great extent govern all practical science, and through it influence man, both morally and physically. The engineer designing a suspension bridge in prayerful reliance on the Holy Spirit's guidance, and finding that the law of the equation last discussed,

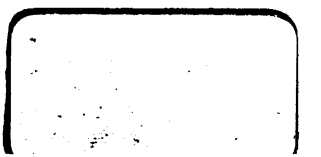
$$y = ax^2 + c\sqrt{x}\,\sin bx,$$

represents such a practical matter as the vibration of the chains under wind pressure, is no more a visionary for applying the analogies of the curve to his own religion and conscience, than for applying the abstract law to determine the strains on the iron windstays of the bridge. The admission is readily made, that such a mathematical line of thought and argument may not practically appeal to many

minds. But where the sceptic recognises no neutral ground, and assumes that what is undefended is indefensible, the believer is bound, if possible, to show that abstract mathematical reasoning supports faith rather than disbelief.

The second objection to a mathematical view of religion is almost a matter of sentiment, a shrinking on the part of many Christians, which is perhaps only of modern growth, from applying any sort of mathematical restrictions, any mechanical attributes, to the Spirit of God, which, however, reveals itself so constantly, as the very essence of mechanical law and order: as the Engineer, "spreading out the heavens, and making Arcturus, Orion, and the Pleiades;" as the Spirit of the Lord, "filling Bezaleel, in the cutting of stones, and in carving of timber, to work in all manner of workmanship;" as the God incarnate in a carpenter's Son. What mysteries and depths of mathematical and mechanical law, what sublime abundance of systems of calculation, must, from everlasting, have existed in the constructive Spirit, which in revealing to man the highest calculus conceivable to human intellect, has as yet but lifted the corner of the veil which conceals the future of mathematical and scientific knowledge! It is but a timid Christianity that would dread mathematical reasoning as applied to the system of work of the great Engineer in the moral, as well as in the physical world; that would assign bounds to the speculations of science, if they be guided by that Holy Spirit, which is as infinitely

great as it is infinitely small, which inspired St. Paul, and taught Newton and Faraday, but which comes as readily

> "A gracious willing Guest
> While He can find one humble heart
> Wherein to rest."

MUIR, PATERSON AND BRODIE, PRINTERS, EDINBURGH.